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TURBULENT FREE CONVECTION IN A VERTICAL LAYER UNDER UNSTABLE STRATIFICATION CONDITIONS

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In a vertical closed layer in which plane isothermal surfaces have different temperatures, a circulation flow is set up, rising on the hot-exchange surface and descending on the cold heat-exchange surface. The temperature outside the boundary layers (in the central vertical section y = 0.5h, where y is the coordinate normal to the heat-exchange surface, and h is the width of the layer) increases linearly as the height of the layer increases, i.e., flow occurs when there is stable stratification of the liquid outside the boundary layer [1-3]. This vertical layer is of interest from the point of view of investigating the structure of a turbulent free-convective boundary layer when there is stable stratification of the liquid outside the layer.

The results of an experimental investigation of the structure of thermal gravitational flows in a vertical layer of liquid, closed and isolated at the top and bottom ends, were presented in [1, 2] (measurements of the mean temperature and velocity), [2, 3] (values of the root mean square velocity and temperature pulsations), and [3] (the probability distribution of the velocity and temperature pulsations and the coefficients of excess and asymmetry).

Investigations of the frequency spectra of the temperature pulsations in a vertical layer at the hot heat-exchange surface in the transition from laminar to turbulent flow, and also when there is a developed turbulent flow in thermal gravitational convection are described in [4].

The spectra of the temperature pulsations, and also the first and second single-point moments in a vertical layer of liquid (the Prandtl number $Pr = \nu/a = 15$, a is the thermal diffusivity, and ν is the coefficient of kinematic viscosity) were obtained in [5] by a numerical solution of the two-dimensional nonstationary Navier–Stokes equations for the turbulent mode of thermal gravitational convection.

In the present paper we describe investigations of the flow structure, the mean values of the temperature, the heat fluxes, and the root mean square temperature pulsations in a vertical plane layer in which the bounding surfaces have the same temperatures and there is a constant temperature gradient along the layer. The temperature on the bounding surfaces decreases with height, as a result of which conditions for unstable stratification are set up close to the surface.

Thermal gravitational flows in vertical layers when there is unstable stratification are observed in vertical cylindrical autoclaves when growing crystals by the hydrothermal method. The basis of this method is the fact that certain materials that are insoluble in water under normal conditions, become soluble at pressures of the order of 1500 atm and temperatures of the order of 600°C. The hydrothermal method is used to grow high-quality crystals, similar in their properties to natural crystals, of such minerals as quartz, beryl, corundum, topaz, etc.

A sketch of the experimental equipment and the nature of the flows is shown in Fig. 1. The working volume was a layer of distilled water 1 with dimensions of 21 × 65 × 300 mm. The plane vertical layer was bounded by brass plates 2 with dimensions of 25 × 65 × 412 mm. The vertical ends of the layer were made of optically transparent plates of plexiglass; the lower end of the layer was closed with a brass plate 3. The ends were attached hermetically to brass plates. A massive copper bar with an electrical heating resistance 4 mounted in it was attached to the lower plate. The temperature of the heater was controlled by means of a PIT-3 regulator and was monitored with a chromel-alumel thermocouple. This heater enabled a specified temperature to be maintained at the lower ends of the heat-exchanging plates.

Cooling channels 5 were drilled at the upper ends of the plates, through which thermostatically controlled water was circulated. In this way a constant temperature was maintained at the upper ends of the plates, which was below the temperature of the heater.

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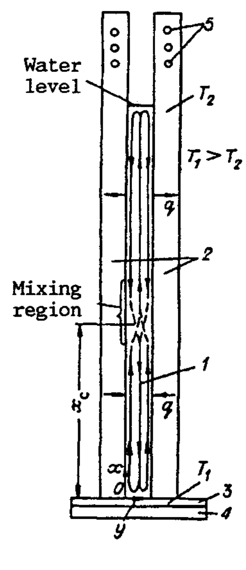

Fig. 1

By heating the plates from below and cooling them on the upper ends a temperature gradient was set up in the heat-exchanging plates $\partial T/\partial x < 0$.

On the heat-exchange surfaces the same temperature gradient was produced, and in the region of the heat-exchange surface unstable stratification conditions were produced in the water layer.

To monitor the temperature on the heat-exchange surface, in the brass plates which bounded the vertical layer, five chromel–alumel thermocouples (0.1 mm) were mounted each a distance of 0.5 mm from the boundary with the liquid, which, using a flat switch, were connected by compensation leads to a zero-thermostat and then to an F4833 digital recording voltmeter. The temperature in the vertical layer of liquid was measured by a nichrome-constantan thermocouple (0.065 mm). The thermocouple pickup was introduced through the free vertical end. The thermocouple pickup was displaced in a fixed horizontal plane using a precision coordinate system, fastened to the upper ends of the brass plates.

The apparatus, which consisted of a CVK-3 microcomputer, an IBM PC/AT-286, a CAMAC, an F4833 analog–digital converter, and the drive of an ENDIM plotter, enabled us to automate the experiment, to carry out the experiments in the necessary real time, to record the instantaneous values of the emfs of the thermocouples with a specified sampling frequency, and to obtain the first and second statistical moments.

In this research we determined the following selected statistical characteristics of samples of the instantaneous values of the temperature, which are considered to be samples of stationary random processes: the mean $T = \sum_{i=1}^{N} T_i/N$, and the root mean square values of the pulsations $(\overline{T'^2})^{1/2} = \left[\sum_{i=1}^{N} (T_i - T)^2\right]^{1/2}$, where $T_i = U_i/R$ is the instantaneous (the actual) value of the temperature, U_i is the measured instantaneous value of the emf of the thermocouple, and R is the sensitivity of the thermocouple, μV/°C.

The thermocouple pickup was displaced in the experimental volume by the drive system of the ENDIM plotter in accordance with a computer program. At each fixed point the emf of the thermocouple was recorded with a sampling frequency of 0.1-2 sec, and the mean values were then calculated at certain intervals of time.

The temperature profiles were measured over the thickness of the layer in 10 horizontal sections at the following heights (from the bottom of the liquid layer): 8.5, 71.5, 96.75, 122, 172.5, 197.75, 223, 248.25, 273.5, and 298.75 mm. The soldered joint of the thermocouple was moved from the heat-exchange surface to the middle of the layer, since the temperature profile in the second half is symmetrical. The step along the horizontal in the survey was variable: up to 1 mm in steps of 0.1 mm,

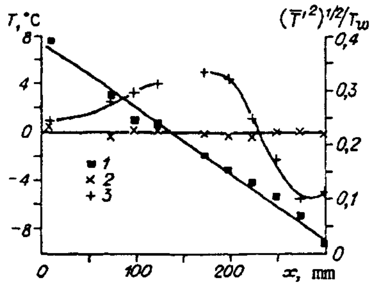

Fig. 2

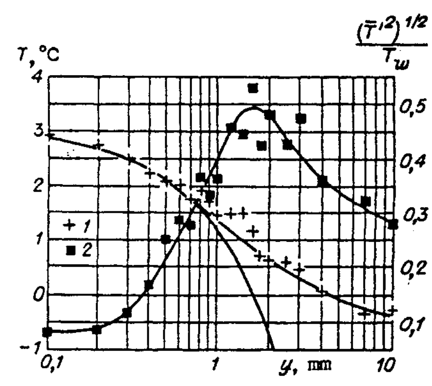

Fig. 3

from 1 to 2 mm in steps of 0.2 mm, and then at distances of 2.5, 3, 4, 7, and 10 mm from the wall. The time taken to make a plot of the actual temperatures at each point up to y = 1 mm was 2-3 min with a sampling frequency of 0.3 sec, and for the external region of the boundary layer the scanning time was 4-6 min with a sampling frequency of 0.6-0.9 sec, respectively.

The experiments were carried out as follows. The equipment was filled with distilled water to a level of 305 mm from the bottom of the layer. The temperature of the bottom of the vessel was constant during the whole time of the experiment. The temperature of the cooling water was 15°C. The measurements were made after steady heat-exchange conditions had been established, which occurred six h after operation of the equipment.

Figure 2 shows the results of measurements of the temperature on the heat-exchange surface T_w and the change in the average local temperature over the height of the layer (points 1 and 2, respectively) in the vertical section y = 0.5h. We took the mean temperature $T_m = 0.5\ (T_1 - T_2)$ as the reference temperature, where T_1 is the temperature of the heat-exchange surface at x = 0, and T_2 is the temperature of the heat-exchange surface at x = H (H is the height of the layer). As follows from Fig. 2, the mean values of the temperature for different x in the section y = 0.5 h are equal to the mean temperature T_m, and hence $T_0 = 0$. Figure 2 also shows the root mean square temperature pulsations 3 outside the boundary layer in the section y = 0.5h.

The temperature of the heat-exchange surface falls as the height h increases, and the conditions for unstable stratification with height $\partial T_w/\partial x < 0$ exist in the region of the heat-exchange surface. Different heat-exchange conditions are produced in the upper part ($x > x_c$) and the lower part ($x < x_c$) over the height of the layer: the heat flux is directed upward from the liquid to the wall, while in the lower part it is directed from the wall to the liquid. The coordinate x_c corresponds to the height where there is no temperature drop $\Delta T = T_w - T_0 = 0$, i.e., $T_w = T_0$ (see Figs. 1 and 2). Since $T_0 = 0$, the temperature difference $T_w - T_0$ for any value of x corresponds to the temperature drop observed in the turbulent boundary layer. The absolute value of $T_m = 40°C$ in all the experiments.

There is a rising flow in the lower part ($x < x_c$) and a descending flow in the upper part of the layer ($x > x_c$) on the heat-exchange surface in the boundary layers (see Fig. 1), and in the neighborhood of x_c these fluxes mix.

We visualized the flow using a shadow pattern and aluminum particles of dimensions 10-20 μ, situated in the water layer. The particles in the flow were visualized by illuminating the layer with a narrow beam of light in the vertical plane, normal to the heat-exchange surface. The turbulent nature of the flow can be observed in the boundary layers on the walls. In the upper part of the layer ($x > x_c$) the average flow in the boundary layers on the heat-exchange surface is descending, while in the center of the layer it is ascending (see Fig. 1). The thickness of the boundary layer is much less than the thickness of the liquid layer, and consequently the average flow velocity on the walls is greater than the velocity of the ascending flow in the center of the layer over its thickness.

In the neighborhood of x_c there is no interface between the flows above and below x_c, but there is a zone where the oppositely traveling fluxes mix. Visual observations indicated periodically occurring rising plumes (approximately every three minutes) along some of the walls in the mixing zone. In the time intervals between the ascensions of the plumes chaotically crossing fluxes are formed in the mixing zone, which are directed opposite to one another downward from the walls to the center of the layer and upward from the walls to the center, the velocities of which gradually decrease until the next plumes break through.

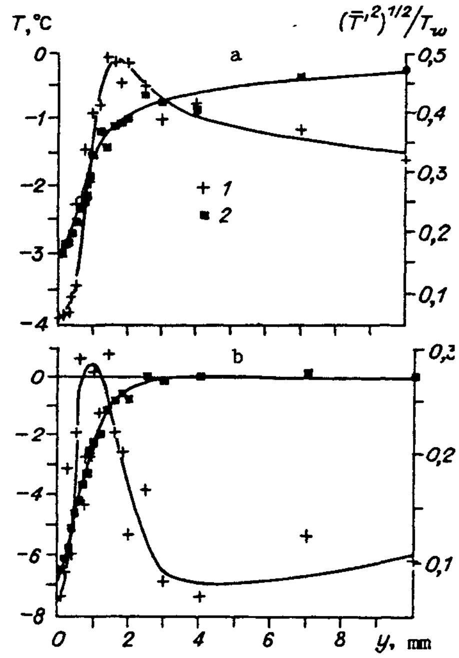

Fig. 4

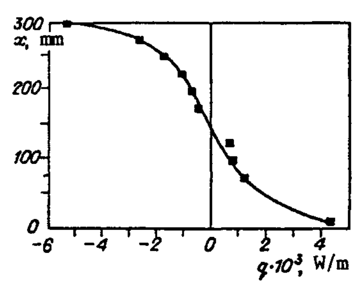

Fig. 5

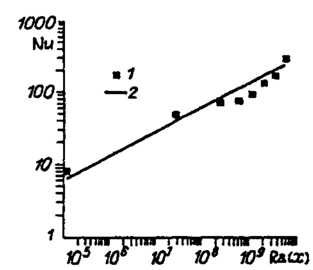

Fig. 6

In Figs. 3 and 4a, b we show profiles of the mean temperature (points 1) and the root mean square values of the temperature pulsations (points 2) for x = 71.5, 197.75, and 273.5 mm, respectively. In the region of the laminar sublayer ($y < \delta_l$) there is a linear variation of the mean temperature from y, and hence the specific thermal flux q was determined from the temperature gradient in it $(\partial T/\partial y)_w$: $q = -\lambda(\partial T/\partial y)_w$ (λ is the thermal conductivity of the liquid).

Figure 5 shows the change in the specific thermal flux on the wall for different heights x. It can be seen that the amount of heat transferred from the wall to the liquid in the lower part of the layer ($x < x_c$) is equal to 16.5 W, while the amount of heat transmitted to the liquid by the wall in the region $x > x_c$ is equal to 17.8 W. The differences in the measured heat fluxes in the upper and lower parts are due to the accuracy of the measurements. The heat losses on the upper free end of the liquid, in all probability, are small, since a considerable fraction of the transmitted heat on the end of the surrounding air would reduce the amount of heat transmitted from the liquid to the wall in the upper part of the layer ($x > x_c$) compared with the heat transmitted from the wall to the liquid in the lower part of the layer ($x < x_c$), whereas the measured heat flux in the upper part is greater than in the lower part.

As can be seen from Figs. 1, 2, and 5, x_c is less than H/2. This is due to the considerable heat-exchange intensity on the lower and upper ends of the layer. On the lower end of the layer the heat-exchange intensity is much greater than at the upper free heat-exchange boundary, which can be regarded as thermally insulated. This is also indicated by the value of the relative root mean square temperature pulsation, which is equal to 0.24 on the lower end and 0.09 on the upper end (see Fig. 2).

TABLE 1

x, mm	T, °C	δ_1, mm	Gr_1
8,5	6,6	0,9	22
71,5	1,6	0,9	5,3
172	−1	1,6	18
197,75	−1	1,5	16
223	−2,35	1,2	18,6
248,25	−2,4	1,3	24,6
273,5	−5	1,1	30,2
298,75	−8	0,9	26,8

In Fig. 6 we show the Nusselt number $Nu_x = qx/\lambda T$ as a function of the Rayleigh number $Ra_x = \beta g \Delta T x^3/a\nu$, where β is the coefficient of volume expansion, g is the acceleration due to gravity, and $\Delta T = T_w$. Because of the incorrectness in determining Nu for small values of the heat fluxes and small temperature drops in the neighborhoods of the mixing region, two points from this region are now shown in Fig. 6. As in the case of turbulent flow, degeneracy of the effect of the linear parameter x: $Nu_x = 0.1Ra_x^{1/3}$ is observed on the isothermal heat-exchange surface under conditions of neutral and stable stratification of the liquid outside the boundary layer [2, 3]. The specific heat flux is found from the relation $q = 0.1\lambda T^{1/3} \cdot (\beta g/a\nu)^{1/3}$ and is independent of x. A similar behavior for the specific heat flux is also characteristic in the horizontal layer of liquid, heated from below under turbulent flow conditions.

Experiments were carried out for a relative height $H/h = 14.3$. The question arises whether it is possible for several mixing zones x_c to occur for large values of H/h. A mixing zone when there is unstable stratification of the liquid at the walls over the height of the layer is due to the existence of a region in which the wall temperature is equal to the mean temperature of the liquid, and the average heat flux at x_c is zero. When there is a monotonic reduction in the wall temperature as the height increases and $\partial T/\partial x = \text{const}$, only one value of x_c is possible where $T_w = T_0$. Hence, when there is unstable stratification in the layer, only a single mixing region x_c is possible for different relative heights of the layer as the turbulent mode develops.

We determined the thickness of the viscous sublayer δ_1 from the deviation from linear of the temperature variation as a function of y. It is more convenient to determine δ_1 from graphs in which the y coordinate is represented in logarithmic form (see Fig. 3). The table shows values of the thickness of the viscous layer δ_1, the temperature drop over the thickness of the viscous sublayer $\Delta T_1 = (T_w - T_1)$, and the Grashof number of the viscous sublayer $Gr_1 = \beta g \Delta T_1 \delta_1^3/\nu^2$ (ΔT_1 is the temperature on the boundary of the viscous sublayer). When there is neutral stratification and there is a constant value of the temperature drop in the boundary layer $Gr_1 = 60/Pr^{1/2}$ [6]. For water, and under our experimental conditions, $Pr = 5.4$, and it therefore follows from the above relation that $Gr_1 = 25.8$.

As can be seen from the table, the order of magnitude of Gr_1 corresponds to the value $Gr_1 = 25.8$, calculated for a constant temperature drop in a turbulent boundary layer over the height. The differences are due to the existence in the external part of the boundary layer of temperature pulsations of considerable intensity. The intensity of the turbulent pulsations can be characterized by the root mean square temperature pulsations, which are shown in Figs. 2-4.

The highest value of $(\overline{T'^2})^{1/2}/T_w$ on the boundary of the viscous sublayer (see Figs. 3 and 4), equal to 0.4-0.55, is a characteristic feature. In the external part of the boundary layer the root mean square value of the temperature pulsations is not zero, as is observed under conditions of neutral and stable stratification in the external part of the boundary layer, and $T_w = \text{const}$ [2, 6]. According to Fig. 2, as one approaches the region x_c where the lower and upper fluxes mix, the intensity of the temperature pulsations in the external part of the boundary layer increases to $(\overline{T'^2})^{1/2}/T_w = 0.35$. When there is stable stratification outside the boundary layer [3] the greatest relative value of the root mean square temperature pulsations (0.15) is observed on the boundary of the viscous sublayer, while in the external part of the boundary layer it approaches zero. Hence, because of the increase in turbulence in the lower part of the boundary layer the thickness of the viscous sublayer decreases as one approaches the mixing zone (see the table).

When there is neutral stratification in the external part of the boundary layer when $T_w = \text{const}$, a turbulent mode begins to develop when $Ra_x = 10^{10}$. In our experiments turbulent flow occurred in the boundary layer even when $Ra_x = 10^5$-10^9. This can also be explained by the condition for unstable stratification of the liquid in the boundary layer. Hence, hydrodynamics and heat exchange when there is unstable stratification over the height of the vertical layer introduce important features into the structure of thermal gravitational flows, giving rise to turbulent flow at relatively low values of Ra_x. A turbulent boundary layer then develops on the walls when there is intense turbulence outside the boundary layer.

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